EFFECT OF GAS COMPRESSIBILITY ON THE STABILITY OF A BOUNDARY LAYER ABOVE A PERMEABLE SURFACE AT SUBSONIC VELOCITIES

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In the present study we investigate the stability of a boundary layer for the condition that the velocity perturbations at the permeable surface are nonzero. The stability for the boundary layer of an incompressible liquid in such a formulation was considered in [1]. For the case of subsonic velocities the effect of compressibility on the flow inside the boundary layer is weak, and in the present article this effect was neglected. The unsteady flow in narrow pores of a permeable covering depends strongly on the compressibility of the gas. Therefore, in the derivation of the relation connecting the pressure oscillations at the permeable surface with the oscillations of the flow through it, the effect of the compressibility was taken into consideration. It is shown that the boundary conditions, and therefore also the stability of the boundary layer at the permeable surface, depend considerably on the Mach number, even for a subsonic exterior flow.

1. The stability of a boundary layer of a compressible liquid at subsonic velocities above an impermeable surface was investigated in [2, 3]. It was shown that the characteristics of the stability over a thermally insulated surface depend weakly on the Mach number. This is explained, on the one hand, by the fact that in the absence of heat exchange the distribution of the mean velocity in the boundary layer of subsonic velocities differs weakly from the velocity distribution for $M=0$, the value of the temperature over the entire layer being approximately constant and equal to the temperature at the outer boundary of the boundary layer [4], and, on the other hand, by the fact that the temperature perturbations in the boundary layer can be neglected [2].

Therefore, in the absence of heat exchange at subsonic velocities the distribution of the perturbation amplitude of the stream function $\Psi=\varphi(y)\exp[i\alpha(x-ct)]$ approximately satisfies the Orr–Sommerfeld equation

$$(U-c)(\varphi''-\alpha^2\varphi)-U''\varphi=\frac{1}{i\alpha\,\mathrm{Re}}(\varphi^{\mathrm{IV}}-2\alpha^2\varphi''+\alpha^4\varphi). \tag{1.1}$$

The usual notation is used here [2, 4]. Equation (1.1) should be solved with four boundary conditions. According to [1], these conditions are

$$\begin{gathered}\varphi(\infty)=\varphi'(\infty)=0,\\ \varphi'(0)=0,\quad (U'(0)-i\alpha/K)\,\varphi(0)=-\frac{1}{i\alpha\,\mathrm{Re}}\varphi'''(0).\end{gathered} \tag{1.2}$$

The first two conditions are the conditions of damping of the perturbations at infinity, the third is the condition of nonpassage along the surface (the plate is permeable only in the normal direction). The fourth condition is obtained from the equations of motion and the law of permeability,

$$v(0)=-Kp(0). \tag{1.3}$$

Here $v(0)$ and $p(0)$ are dimensionless perturbations of velocity and of pressure in the boundary layer near the surface of the permeable plate; K is a coefficient of proportionality, determined below. The pressure

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is referred to the total head at the boundary of the boundary layer ρU_e^2, and the velocity is referred to the velocity at the boundary of the boundary layer U_e. To determine K we consider the model proposed in [1]. The pores of the permeable plate have a cylindrical shape and are oriented in the normal direction to the plate surface. Thus, we consider a perforated plate with small diameters of the holes and distances between them, at least in comparison with the thickness of the boundary layer. The dimensions of the holes (pores) are large enough so that we can take the pressure distribution in them to be independent of the radial direction. On the basis of this model, an expression for K was obtained in [1] for an incompressible liquid.

For the case of a compressible gas, in order to determine K we should use the theory of propagation of acoustic waves in long narrow channels. The propagation of acoustic waves is characterized by a propagation constant λ and characteristic impedance Z_0. By analogy with the electrically conducting line in [5] we obtain values of λ and Z_0, expressed in terms of acoustic parameters: Z, the impedance of an element of pipe, and Y, a coefficient characterizing the margin of energy of compression and loss of thermal energy of a tube element due to heat transfer to the walls of the pipe. The acoustic parameters characterize the relation between the bulk velocity and pressure. In the present study it is convenient to consider the relation between the velocity V_1 and the pressure averaged over the pipe cross section. For a law of propagation of velocity and pressure averaged over the cross section along a long pipe we evidently can use an analogy with an electrically conducting line having impedance SZ and a second parameter Y/S, where S is the cross-sectional area of the pipe. Using Z and Y, taken from [6], we can write the dimensionless quantities $Z_1 = Z \cdot S\delta/\rho U_e$ and $Y_1 = Y \cdot \rho U_e \delta/S$ as

$$Z_1 = i\alpha c \frac{I_0(\sqrt{i\alpha c \operatorname{Re}}\, r_1)}{I_2(\sqrt{i\alpha c \operatorname{Re}}\, r_1)}, \qquad (1.4)$$

$$Y_1 = -i\alpha c M_0^2 \left[\varkappa + (\varkappa - 1)\frac{I_2(\sqrt{i\alpha c \operatorname{Re}\,\sigma}\, r_1)}{I_0(\sqrt{i\alpha c \operatorname{Re}\,\sigma}\, r_1)}\right].$$

As dimensional quantities we use the quantities earlier assumed in the present study. It was taken into account that the frequency $\omega = -\alpha c U_e/\delta$, where δ is the thickness of the boundary layer. In (1.4) we assume the following notation: M_0 is the ratio of the flow velocity at the external boundary of the boundary layer to the acoustic velocity near the surface, σ is the Prandtl number, $\varkappa$ is the adiabatic exponent, r_1 is the ratio of the radius of a hole (pore) to the thickness of the boundary layer, and I_0 and I_2 are Bessel functions of order zero and order two. We should note that in the present study we consider cases in which the temperature distribution over the layer is approximately constant; therefore, by M_0 we shall mean the Mach number of the incoming flow. By analogy with [5] the propagation constant λ and the characteristic impedance Z_0 are defined by the equations

$$\lambda = (Z_1 Y_1)^{1/2}, \quad Z_0 = Z_1/\lambda.$$

On one end of the pipe (pore) let us be given the relation

$$p(-H) = X_1 \cdot v_1(-H). \qquad (1.5)$$

Then by analogy with the results presented in [7] we can obtain

$$\frac{v_1(0)}{p(0)} = -\frac{1}{Z_0}\frac{Z_0 - X_1 \operatorname{th}(\lambda H)}{Z_0 \operatorname{th}(\lambda H) - X_1}.$$

If the fraction of the surface occupied by the holes is n, then the velocity near the surface $v(0) = nv_1(0)$; therefore,

$$K = -\frac{v(0)}{p(0)} = \frac{n}{Z_0}\frac{Z_0 - X_1 \operatorname{th}(\lambda H)}{Z_0 \operatorname{th}(\lambda H) - X_1}. \qquad (1.6)$$

The value of X_1 is determined simply if condition (1.5) is written for the end of the pipe (pore) adjacent to a large volume in which the gas has no average motion (e.g., a chamber of weak suction). Using the equation $v_1(-H) = v(-H)/n$ (n is the porosity, H is the thickness of the permeable covering, and $v(-H)$ are the velocity perturbations near the permeable covering), according to [4], we can obtain

$$X_1 = X/n = (i\alpha c - \alpha^2/\operatorname{Re})\ (\alpha + \gamma)/\alpha\gamma n,$$

$$\gamma = -\sqrt{-i\alpha c \operatorname{Re} + \alpha^2}.$$

2. In certain particular cases we can qualitatively explain the effect of permeability of perturbations through the surface on the stability of the boundary layer.

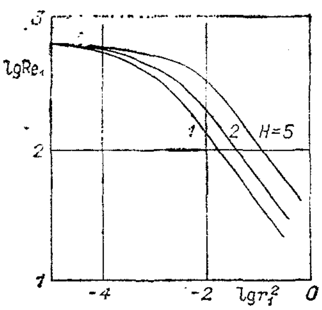

Fig. 1

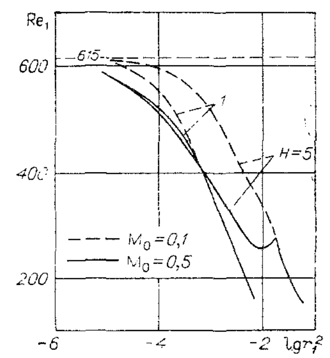

Fig. 2

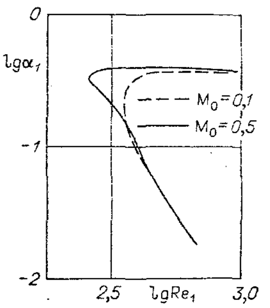

Fig. 3

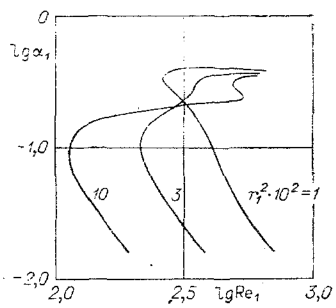

Fig. 4

a) If the liquid is incompressible ($M_0 = 0$), then we should consider the limiting value of K as $|\lambda H| \to 0$. In this case

$$K = -Q(1+XQ)^{-1}, \tag{2.1}$$

where $Q = -n/Z_1H$. Equation (2.1) agrees with the equation obtained earlier in [4]. We see that K depends only on two parameters of the permeable plate: n/H and r_1^2. If the diameters of the holes are so small that $|\sqrt{i\alpha c \mathrm{Re}}\, r_1| \ll 1$, then we can easily show that $K = nr_1^2\mathrm{Re}/8H$, i.e., K depends on the single parameter nr_1^2/H.

b) For a compressible liquid ($M_0 \neq 0$), the value of $|\lambda H|$ can assume large values for moderate Mach numbers but sufficiently large pores (holes). For $|\lambda H| \gg 1 \mathrm{th}(\lambda H) = 1$, therefore, $K = n/Z_0$ or

$$K = n\lambda/Z_1. \tag{2.2}$$

The expression obtained is independent of the thickness of the permeable covering. This indicates that with increasing $|\lambda H|$ the stability characteristics will approach certain finite limits. We see that these limits will depend on the parameters of the permeable covering r_1^2 and n, and on the numbers M_0, σ, and $\varkappa$. If $|\sqrt{i\alpha c \mathrm{Re}}\, r_1| \ll 1$, then we can show that K depends only on the parameter $nM_0r_1^2$.

From condition (1.3) it follows that the energy flux through the permeable surface

$$N = \frac{p(0)\bar{v}(0) + \bar{p}(0)v(0)}{4} = |p(0)|^2 \operatorname{Re}(K).$$

We can show that for small diameters of the pores both for an incompressible liquid ($|\lambda H| = 0$) and also for a compressible liquid for $|\lambda H| \gg 1$, the real part of K is positive; therefore $N > 0$. Thus, the energy decreases in the boundary layer and by analogy with the definition of [8] for elastic coverings the permeable covering can be assumed passive. In [8] it was indicated that in the majority of cases the stability of the boundary layer for passive coverings is reduced in comparison with the stability for a rigid impermeable surface. Therefore, we should expect a reduction in stability of the boundary layers above permeable surfaces both for condition (2.1) (this is found to be in agreement with the results of [1], obtained for an incompressible liquid) and also for condition (2.2).

In other cases, when it is necessary to use (1.6) to determine K, a simple analysis is impossible, and it is difficult to draw any conclusions without detailed calculation. Therefore, in the present study, besides an approximate analysis we carry out some calculations using (1.6).

3. The characteristics of stability are calculated for a velocity profile, given in the form of a sixth-degree polynomial

$$U=2y-5y^4+6y^5-2y^6,$$

which is a good approximation of the distribution of the longitudinal velocity in the boundary layer for a plane impermeable plate. If the diameters of the holes of a permeable plate are small, and suction is absent, the effect of the holes on the velocity distribution will be weak. Therefore, in the present study such an approximation of the velocity profile is assumed to be suitable in calculations of stability of the boundary layer for a plane permeable surface without suction.

The results given below are obtained on the basis of a numerical solution of Eq. (1.1) with boundary conditions (1.2), in which K is determined by Eq. (1.6). All the calculations are carried out for fixed values of $\sigma=0.72$, $\varkappa=1.4$, and $n=0.5$.

The calculation results are presented on the graphs, where Re_1 and α_1 are the Reynolds number and the wave number plotted over the thickness of the displacement.

In Fig. 1 we show the dependence of the critical Reynolds numbers Re_1^* on r_1^2 for three values of H ($M_0=0.1$). Additional calculations for $M_0=0$ show that for the parameters r_1^2 and H, indicated in Fig. 1, the stability characteristics for $M_0=0.1$ and $M_0=0$ differ weakly from each other. This is explained by the fact that for $M_0=0.1$ $|\lambda H| \ll 1$ and for determination of K we can use Eq. (2.1). For larger values of M_0 the effect of the Mach number on the stability characteristics becomes noticeable (Figs. 2 and 3). From Fig. 2 we see that Re_1^* for $M_0=0.5$ is less than Re_1^* obtained for $M_0=0.1$, i.e., with increasing M_0 the stability is reduced.

For $M_0=0.5$ with $H \geqslant 2$ and $r_1^2 < 10^{-3}$, the thickness of the permeable plate has no effect on the value of Re_1^*, which is found to be in agreement with the conclusions of the preceding section [see (2.2)]. In a certain region of values of r_1^2 depending on M_0 and H (e.g., for $M_0=0.5$, $H=5$ $r_1^2 \approx 10^{-2}$–$3 \cdot 10^{-2}$), we observe a nonmonotonicity (although weak) in the dependence of Re_1^* on r_1^2, i.e., an increase in the diameter of the holes does not always lead to a reduction in the stability of the boundary layer. Undoubtedly, the region of values in which we observe a violation of the monotonicity of the variation of Re_1^* for variation of r_1^2 depends on other parameters (σ, $\varkappa$, and n), which in the present article do not vary, and also on the form of the velocity profile.

A comparison of the curves of neutral stability (Fig. 3) for $M_0=0.1$ and $M_0=0.5$ indicates, on the one hand, the deformation of the curve in the region of Re_1^* with variation of M_0, and, on the other hand, it indicates the coincidence of the branches of these curves. The latter is connected with the fact that on the branches of the curve of neutral stability $|\lambda H| < 1$ and K is determined from (2.1), where the value of M_0 does not appear.

Figure 4 shows the variation in the shape of the curve of neutral stability ($M_0=0.5$, $H=5$) as a function of r_1^2. We see the formation of two extremal values of Re_1 on a single curve, the minimum of which is Re_1^*. The existence of two extremal values of Re_1 is explained by the kink on the curve of Fig. 2 ($M_0=0.5$, $H=5$).

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